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Rigorous derivation of quasi-mutual entropy in Jaynes–Cummings model

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Abstract

The quasi-mutual entropy in the Jaynes–Cummings model is rigorously derived without using the diagonal approximation. The variation of the correlation in this model for the time development and the statistical mixture parameter is discussed.

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1. Introduction

It is known that the time development in the Jaynes–Cummings model (JCM) exhibits correlation between the two-level atom and the field [1]. So we have studied the time development of the final state in the JCM in the previous papers by applying the quasi-mutual entropy with the diagonal approximation [2, 3]. In this paper, we derive the quasi-mutual entropy for the JCM without the diagonal approximation and then discuss the variation of the correlation in this model for the time development and the statistical mixture parameter, as it was pointed out in [4] that the mutual entropy measures the degree of the total of the classical and quantum correlation.

We consider two subsystems $\mathcal{H}_1$ and $\mathcal{H}_2$ represented by Hilbert space. Let $\mathfrak{S}(\mathcal{H}_i)$ $(i = 1, 2)$ be state spaces (the set of all density operators). Also $\mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2)$ denotes the state space in the composite system $\mathcal{H}_1 \otimes \mathcal{H}_2$.

For a quantum state $\varrho \in \mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2)$, the quasi-mutual entropy is defined by the following formula as a distance (difference) from a product state $\mathrm{tr}_{\mathcal{H}_1}\varrho \otimes \mathrm{tr}_{\mathcal{H}_2}\varrho \in \mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2)$:

$$I(\varrho) = \mathrm{tr}\,\varrho\left(\log\varrho - \log\left(\mathrm{tr}_{\mathcal{H}_1}\varrho \otimes \mathrm{tr}_{\mathcal{H}_2}\varrho\right)\right).$$

Here we should note that the term *quasi* means that this mutual entropy does not depend on the channelling transformation describing the physical processes such as communication processes from input to output states. The quasi-mutual entropy was first defined in [5] to be applied to irreversible processes as a consistent work of mathematical formalism based on functional analysis or quantum mechanical entropy theory, whereas the quantum mutual entropy was defined in [6] to be applied to communication processes owing to the remarkable concept of *compound states* which correspond to the joint probability in classical theory.

In section 2, we briefly review the JCM. Also we derive the quasi-mutual entropy in the JCM more rigorously in section 3 than in the previous papers [2, 3] which allow diagonal approximation mentioned in section 3. Finally in section 4 we give some numerical computations for the quasi-mutual entropy in the JCM and discuss some features of the JCM from the viewpoint of the correlation between the atom and the photon.

2. Jaynes–Cummings model

The quantum electrodynamical interaction of a single two-level atom with a single mode of an electromagnetic field is described by the well-known Jaynes–Cummings model [7]. The JCM is the simplest nontrivial model of two interacting fully quantum systems and has an exact solution. It also brings us some interesting phenomena such as collapses and revivals. It has been investigated in detail by many researchers from various points of view. The JCM is not only an important problem in itself but also gives an excellent example of the so-called quantum open system problem [8], namely the interaction between a system and a reservoir. In the previous paper [9], we treated the JCM as a problem in nonequilibrium statistical mechanics and applied quantum mutual entropy [5] based on von Neumann entropy by finding the quantum mechanical channel [5] which expresses the state change of the atom on the JCM. This study was an attempt to obtain a new insight into the dynamical change of the state for the atom on the JCM [9].

On the other hand, this model has one of the most interesting features, which is the correlation developed between the atom and the field during the interaction. There have been several approaches to analyse the time evolution in this model, for instance, von Neumann entropy and atomic inversion. In the previous paper [2], we adopted the quasi-mutual entropy to measure the degree of correlation in the time development of the JCM and showed that the quasi-mutual entropy can be controlled by means of the squeezed state in [3]. In this paper, we rigorously derive the quasi-mutual entropy in the JCM, which takes into account the negative energy and does not allow diagonal approximation.

The resonant JCM Hamiltonian can be expressed by the rotating-wave approximation in the following form:

$$H = H_A + H_F + H_I \qquad H_A = \tfrac{1}{2}\hbar\omega_A\sigma_z$$
$$H_F = \hbar\omega_F a^* a \qquad H_I = \hbar g(a \otimes \sigma^+ + a^* \otimes \sigma^-)$$

where g is a coupling constant, $\sigma^\pm$ are the pseudo-spin matrices of a two-level atom, σ_z is the z-component of the Pauli spin matrix and a and a^* are the annihilation and the creation operators of a photon, respectively. It is almost impossible to physically realize the pure states, so we suppose that the initial states of the atomic system are the statistical mixture states of an excited state and a ground state, which are the more realistic representation of the states. That is, we now suppose that the initial state of the atom is a statistical mixture of the ground state and the excited state:

$$\rho = \lambda_0 E_0 + \lambda_1 E_1 \in \mathfrak{S}_A \tag{1}$$

where $E_0 = |{\downarrow}\rangle\langle{\downarrow}|$, $E_1 = |{\uparrow}\rangle\langle{\uparrow}|$, $\lambda_0 + \lambda_1 = 1$. Let the field initially be in a coherent state:

$$\varpi = |\theta\rangle\langle\theta| \in \mathfrak{S}_F \qquad |\theta\rangle = \mathrm{e}^{-\frac{1}{2}|\theta|^2} \sum_l \frac{\theta^l}{\sqrt{l!}} |l\rangle. \tag{2}$$

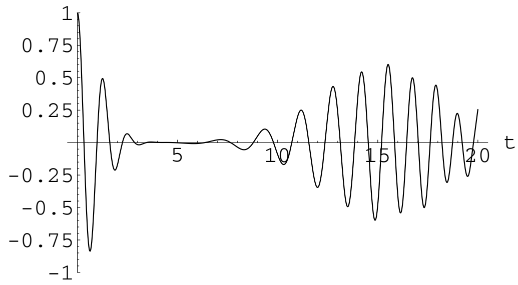

Figure 1. The population inversion as a function of time t.

The continuous map $\mathcal{E}_t^*$ describing the time evolution between the atom and the field for the JCM is defined by the unitary operator generated by the total Hamiltonian H such that

$$\mathcal{E}_t^* : \mathfrak{S}_A \to \mathfrak{S}_A \otimes \mathfrak{S}_F \qquad \mathcal{E}_t^* \rho = U_t (\rho \otimes \varpi) U_t^* \qquad U_t \equiv \mathrm{e}^{-\mathrm{i}tH/\hbar}. \tag{3}$$

This unitary operator U_t is written as

$$U_t = \mathrm{e}^{-\mathrm{i}tH/\hbar} = \sum_{n=0}^{\infty} \sum_{j=0}^{1} \mathrm{e}^{-\mathrm{i}tE_j^{(n)}} \left|\Phi_j^{(n)}\right\rangle \left\langle \Phi_j^{(n)}\right| \tag{4}$$

where

$$E_j^{(n)} = \hbar\omega_F \left(n + \frac{1}{2}\right) + \frac{\hbar}{2} (-1)^{j+1} \sqrt{(\omega_F - \omega_A)^2 + 4\Omega_n^2} \tag{5}$$

are the eigenvalues with $\Omega_n = g\sqrt{n+1}$, called Rabi frequency, and $\left|\Phi_j^{(n)}\right\rangle$ are the eigenvectors associated with $E_j^{(n)}$. When we set $\omega_A = \omega_F$, the transition probability that the atom is initially prepared in the excited state and stays in the excited state after time t is given by

$$c(t) = \mathrm{e}^{-|\theta|^2} \sum_{n=0}^{\infty} \frac{|\theta|^{2n}}{n!} \cos^2 \Omega_n t.$$

Also the transition probability that the atom is initially prepared in the excited state and is in the ground state after time t is given by

$$s(t) = \mathrm{e}^{-|\theta|^2} \sum_{n=0}^{\infty} \frac{|\theta|^{2n}}{n!} \sin^2 \Omega_n t.$$

Note that $c(t) - s(t)$ is often called the population inversion and used to analyse the time development of the atomic system.

Figure 1 shows the population inversion for the coupling constant $g = 1$ and a mean photon number $|\theta|^2 = 5$. This model yields a *dephase* around the time $t_c \approx 1/g$ and shows the damped oscillation with the Gaussian envelope. This damping is caused by the difference and interference of the Rabi frequency Ω_n. This damping phenomenon is often called *Cummings collapses* [1, 10]. Figure 1 shows that the atom in this model is at the most uncertain state at the time when the population inversion is equal to 0, namely around $t \cong 3$–7. So, it can be seen that the system in this model at that time becomes the most correlated. Later the system shows *revival* around $t_r \cong 2\pi|\theta|/g$. The reason for *revival* is considered as a *rephase*, and its revival periodically appears at each $T_k = kt_r$ $(k = 1, 2, 3, \ldots)$. It is also known [1, 11] that the system in this model returns most closely to a pure state of the atom around $t_r/2$, during the collapse interval. For details on this model, the readers may refer to the excellent reviews [1, 12].

3. Rigorous derivation of quasi-mutual entropy in the JCM

The eigenvectors of the total Hamiltonian H are given by

$$\left|\Phi_0^{(n)}\right\rangle = \cos\theta_n|n\otimes\uparrow\rangle - \sin\theta_n|n+1\otimes\downarrow\rangle \tag{6}$$

$$\left|\Phi_1^{(n)}\right\rangle = \sin\theta_n|n\otimes\uparrow\rangle + \cos\theta_n|n+1\otimes\downarrow\rangle \tag{7}$$

where

$$\tan\theta_n \equiv \frac{2g\sqrt{n+1}}{(\omega_F-\omega_A)+\sqrt{(\omega_F-\omega_A)^2+4g^2(n+1)}}.$$

Throughout the present paper, we consider only the case of

$$\omega_A = \omega_F \equiv \omega \tag{8}$$

so we can take $\tan\theta_n = 1$, namely $\theta_n = \frac{\pi}{4}$. Then defining

$$\widetilde{E_{jk}^{(nm)}} \equiv \mathrm{e}^{-\mathrm{i}t\left(E_j^{(n)}-E_k^{(m)}\right)} \qquad (j=0,1 \quad k=0,1)$$

the final state at any time t is given by

$$\begin{aligned}\mathcal{E}_t^*\rho &\equiv U_t(\rho\otimes\varpi)U_t^* \\ &= \sum_{m,n=0}^{\infty}\sum_{j,k=0}^{1}\widetilde{E_{jk}^{(nm)}}\left\langle\Phi_j^{(n)}\right|\rho\otimes\varpi\left|\Phi_k^{(m)}\right\rangle\left|\Phi_j^{(n)}\right\rangle\left\langle\Phi_k^{(m)}\right|\end{aligned} \tag{9}$$

In the previous papers [2, 3], we derived the quasi-mutual entropy for the JCM by applying the following approximation:

$$\left\langle\Phi_j^{(n)}\right|\rho\otimes\varpi\left|\Phi_k^{(m)}\right\rangle \cong \left\langle\Phi_j^{(n)}\right|\rho\otimes\varpi\left|\Phi_k^{(n)}\right\rangle\delta_{mn} \qquad \text{for} \quad j=0,1 \quad k=0,1.$$

However, in the present paper, we rigorously derive the quasi-mutual entropy in the JCM without the above approximation.

Taking into account the negative energy term, $-\frac{1}{2}\hbar\omega_A$, the following eigenequations hold for the total Hamiltonian H:

$$H\left|\Phi_j^{(n)}\right\rangle = E_j^{(n)}\left|\Phi_j^{(n)}\right\rangle \qquad (n=0,1,2,\ldots)$$

$$H|-1\otimes\downarrow\rangle = -\tfrac{1}{2}\hbar\omega_A|-1\otimes\downarrow\rangle$$

where $E_j^{(n)}$ and $\left|\Psi_j^{(n)}\right\rangle$ are given in (5), (6) and (7). Under the assumptions (1), (2) and (8), the final state is derived as follows:

$$\mathcal{E}_t^*\rho = \lambda_0|\Psi_0(t)\rangle\langle\Psi_0(t)| + \lambda_1|\Psi_1(t)\rangle\langle\Psi_1(t)| \tag{10}$$

where

$$|\Psi_1(t)\rangle \equiv \mathrm{e}^{-\frac{1}{2}|\theta|^2}\sum_{j=0}^{1}\sum_{n=0}^{\infty}\frac{\theta^n}{\sqrt{2n!}}\mathrm{e}^{-\mathrm{i}tE_j^{(n)}}\left|\Phi_j^{(n)}\right\rangle$$

$$|\Psi_0(t)\rangle \equiv \mathrm{e}^{-\frac{1}{2}|\theta|^2}\left(\mathrm{e}^{\mathrm{i}\frac{\omega}{2}t}|0\otimes\downarrow\rangle + \sum_{j=0}^{1}\sum_{n=0}^{\infty}\frac{(-1)^{j+1}\theta^{n+1}}{\sqrt{2(n+1)!}}\mathrm{e}^{-\mathrm{i}tE_j^{(n)}}\left|\Phi_j^{(n)}\right\rangle\right).$$

Since von Neumann entropy for the total system does not change for unitary evolution, it is given by

$$S(\mathcal{E}_t^*\rho) = -\sum_{i=0}^{1}\lambda_i\log\lambda_i. \tag{11}$$

Taking the partial trace over the atomic system, we obtain

$$\rho_t^F = \mathrm{tr}_A \mathcal{E}_t^* \rho$$
$$= \lambda_0(|\psi_1(t)\rangle\langle\psi_1(t)| + |\psi_2(t)\rangle\langle\psi_2(t)|) + \lambda_1(|\psi_3(t)\rangle\langle\psi_3(t)| + |\psi_4(t)\rangle\langle\psi_4(t)|)$$

where

$$|\psi_1(t)\rangle = \mathrm{e}^{-\frac{1}{2}|\theta|^2} \sum_{n=0}^{\infty} \frac{\theta^n}{\sqrt{n!}} \mathrm{e}^{-\mathrm{i}\omega_n t} \cos \Omega_n t |n\rangle$$

$$|\psi_2(t)\rangle = -\mathrm{i}\, \mathrm{e}^{-\frac{1}{2}|\theta|^2} \sum_{n=0}^{\infty} \frac{\theta^{n-1}}{\sqrt{(n-1)!}} \mathrm{e}^{-\mathrm{i}\omega_{n-1} t} \sin \Omega_{n-1} t |n\rangle$$

$$|\psi_3(t)\rangle = -\mathrm{i}\, \mathrm{e}^{-\frac{1}{2}|\theta|^2} \sum_{n=0}^{\infty} \frac{\theta^{n+1}}{\sqrt{(n+1)!}} \mathrm{e}^{-\mathrm{i}\omega_n t} \sin \Omega_n t |n\rangle$$

$$|\psi_4(t)\rangle = \mathrm{e}^{-\frac{1}{2}|\theta|^2} \sum_{n=0}^{\infty} \frac{\theta^n}{\sqrt{n!}} \mathrm{e}^{-\mathrm{i}\omega_{n-1} t} \cos \Omega_{n-1} t |n\rangle$$

with

$$\omega_n = \omega\left(n + \tfrac{1}{2}\right) \qquad \Omega_n = g\sqrt{n+1}.$$

Then the von Neumann entropy for the reduced state $S(\rho_t^F)$ is computed by

$$S(\rho_t^F) = -\sum_{i=1}^{4} \lambda_i^F(t) \log \lambda_i^F(t) \tag{12}$$

where $\lambda_i^F(t)$ are the solutions of

$$\det[\rho(\hat{t}) - \lambda(\hat{t})N(\hat{t})] = 0$$

where $\rho(\hat{t})$ and $N(\hat{t})$ are the 4×4 matrices having the following elements:

$$[\rho(\hat{t})]_{ij} \equiv \langle\psi_i(t)|\rho_t^F|\psi_j(t)\rangle \qquad (i, j = 1, 2, 3, 4)$$
$$[N(\hat{t})]_{ij} \equiv \langle\psi_i(t)|\psi_j(t)\rangle \qquad (i, j = 1, 2, 3, 4).$$

On the other hand, the final state of the atomic system is given by taking the partial trace over the field system:

$$\rho_t^A \equiv \mathrm{tr}_F \mathcal{E}_t^* \rho$$
$$= c_{\uparrow\uparrow}(t)|\uparrow\rangle\langle\uparrow| + c_{\uparrow\downarrow}(t)|\uparrow\rangle\langle\downarrow| + c_{\downarrow\uparrow}(t)|\downarrow\rangle\langle\uparrow| + c_{\downarrow\downarrow}(t)|\downarrow\rangle\langle\downarrow|$$

where

$$c_{\uparrow\uparrow}(t) = \mathrm{e}^{-|\theta|^2} \sum_{n=0}^{\infty} \left\{ \lambda_0 \frac{|\theta|^{2(n+1)}}{(n+1)!} \sin^2 \Omega_n t + \lambda_1 \frac{|\theta|^{2n}}{n!} \cos^2 \Omega_n t \right\}$$

$$c_{\downarrow\uparrow}(t) = c_{\uparrow\downarrow}(t)^*$$

$$c_{\uparrow\downarrow}(t) = \mathrm{i}\, \mathrm{e}^{-(|\theta|^2 + \mathrm{i}\omega t)}$$
$$\times \sum_{n=0}^{\infty} \left\{ \lambda_1 \frac{\theta^n \theta^{*(n-1)}}{\sqrt{n!(n-1)!}} \cos \Omega_n t \sin \Omega_{n-1} t - \lambda_0 \frac{\theta^{n+1} \theta^{*n}}{\sqrt{(n+1)!n!}} \sin \Omega_n t \cos \Omega_{n-1} t \right\}$$

$$c_{\downarrow\downarrow}(t) = \mathrm{e}^{-|\theta|^2} \sum_{n=0}^{\infty} \left\{ \lambda_0 \frac{|\theta|^{2n}}{n!} \cos^2 \Omega_{n-1} t + \lambda_1 \frac{|\theta|^{2(n-1)}}{(n-1)!} \sin^2 \Omega_{n-1} t \right\}.$$

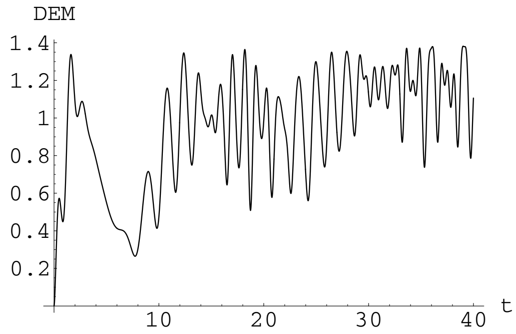

Figure 2. The quasi-mutual entropy as a function of t in the case of $\lambda_0 = 0.0$, $\omega = 1$, $g = 1$ and $|\theta|^2 = 5$.

Then the von Neumann entropy for the reduced state $S(\rho_t^A)$ is computed by

$$S(\rho_t^A) = -\sum_{i=1}^{2} \lambda_i^A(t) \log \lambda_i^A(t) \tag{13}$$

where $\lambda_i^A(t)$ is given by

$$\lambda_i^A(t) = \tfrac{1}{2}\left\{1 + (-1)^i \sqrt{(c_{\uparrow\uparrow}(t) - c_{\downarrow\downarrow}(t))^2 + 4|c_{\uparrow\downarrow}(t)|^2}\right\}.$$

Thus we rigorously obtain the quasi-mutual entropy in the JCM from (11), (12) and (13):

$$\begin{aligned} I(\mathcal{E}_t^*\rho) &\equiv \operatorname{tr} \mathcal{E}_t^*\rho\big(\log \mathcal{E}_t^*\rho - \log\left(\rho_t^A \otimes \rho_t^F\right)\big) \\ &= S(\rho_t^A) + S(\rho_t^F) - S(\mathcal{E}_t^*\rho) \qquad (14) \\ &= -\sum_{i=1}^{4} \lambda_i^F(t) \log \lambda_i^F(t) - \sum_{i=1}^{2} \lambda_i^A(t) \log \lambda_i^A(t) + \sum_{i=0}^{1} \lambda_i \log \lambda_i. \qquad (15) \end{aligned}$$

4. Numerical computations

We should note that if we set the statistical mixture parameter of the initial state in the atomic system $\lambda_0 = 0$ or 1, the final state presented in (10) becomes the pure state and then $S(\mathcal{E}_t^*\rho) = 0$. Therefore it is sufficient to use von Neumann entropy in order to measure the degree of correlation for the above cases. Then our quasi-mutual entropy takes just twice the reduced von Neumann entropy, i.e., $I(\mathcal{E}_t^*\rho) = 2S(\rho_t^A)$. In paper [13], these situations have been considered and the reduced von Neumann entropy has been applied to analyse the quantum fluctuations; however, it does not directly focus on the degree of the correlation. In the general case (i.e., $\lambda_0 \neq 0$ or 1), the final state does not necessarily become the pure state, so that we need to adopt the quasi-mutual entropy in order to measure the degree of correlation in the JCM. In fact, the initial state of the atomic system was considered as an exited state or a ground state in [13]; however, we start from the statistical mixture of an exited state and a ground state, namely equation (1) as an initial state of the atomic system. Thus, our initial setting enables us to discuss the variation of the quasi-mutual entropy by the difference of the statistical mixture parameter λ_0 from the initial atomic system.

Figures 2–5 show the time development of the quasi-mutual entropy in the case of $\lambda_0 = 0$, $\lambda_0 = 0.1$, $\lambda_0 = 0.3$ and $\lambda_0 = 0.5$. From these figures, we find that the amplitude of the oscillation decreases with increasing λ_0. We find that our quasi-mutual entropy in figure 2

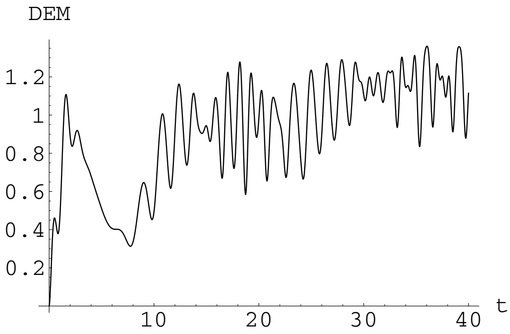

Figure 3. The quasi-mutual entropy as a function of t in the case of $\lambda_0 = 0.1$, $\omega = 1$, $g = 1$ and $|\theta|^2 = 5$.

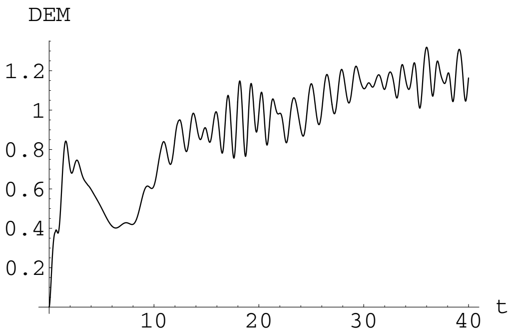

Figure 4. The quasi-mutual entropy as a function of t in the case of $\lambda_0 = 0.3$, $\omega = 1$, $g = 1$ and $|\theta|^2 = 5$.

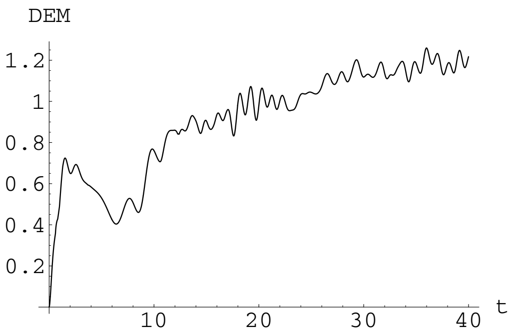

Figure 5. The quasi-mutual entropy as a function of t in the case of $\lambda_0 = 0.5$, $\omega = 1$, $g = 1$ and $|\theta|^2 = 5$.

($\lambda_0 = 0$) takes just twice the value of von Neumann entropy plotted in [13] for all t. This result is quite natural, because $\mathcal{E}_t^* \rho$ is a pure state in this case ($\lambda_0 = 0$). In the short time of the figures (figures 2–5), we also find that the quasi-mutual entropy takes the local maximum points at around $t \cong 2$ and they become smaller as λ_0 increases.

Figure 6 shows the quasi-mutual entropy as a function of statistical mixture parameter λ_0 at the first revival time $t_r \cong 2\pi|\theta|/g$. From this figure, we find that the quasi-mutual

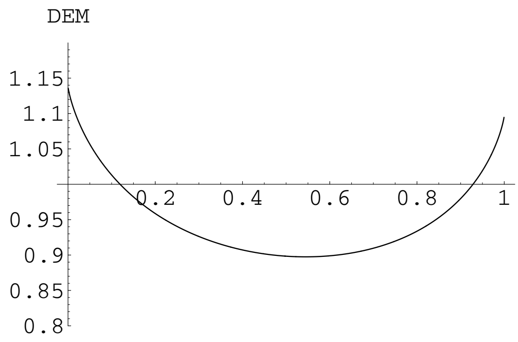

Figure 6. The quasi-mutual entropy for λ_0 in the case of $t = t_r$, $\omega = 1$, $g = 1$ and $|\theta|^2 = 5$.

entropy takes the largest value in $\lambda_0 = 0$ which is one of the pure states. Although it might be expected that the quasi-mutual entropy has an axial symmetry with respect to $\lambda_0 = 0.5$ (as the quasi-mutual entropy does in $t = 0$), figure 6 shows that the quasi-mutual entropy does not have quite an axial symmetry with respect to $\lambda_0 = 0.5$. We can also say that the quasi-mutual entropy has a different value even for pure states ($\lambda_0 = 0$ or $\lambda_0 = 1$). If we treat the time development of the JCM as a kind of physical transformation, the degree of correlation cannot keep a symmetry for $\lambda_0 = 0.5$ by this transformation. This result is caused by the fact that, according to the initial state of the two-level atom being an upper level or a lower level, it exchanges a different energy with the field (periodically the lower-level atom absorbs a photon and the upper-level atom emits a photon) in the time development of the JCM. These results could be obtained by setting the initial state of the atom as the statistical mixture of an exited state and a ground state.

As we have seen, we have rigorously derived a kind of measure of the degree of correlation for the final state in the JCM without diagonal approximation. We suggest that the method using the quasi-mutual entropy can be applied to the other Hamiltonian models such as the Raman-coupled model.

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